

Development of Multidimensional MHD Model for the Solar Corona and Solar Wind

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Abstract. We are developing a time stationary self-consistent 2D MHD model of the solar corona and solar wind that explicitly solves the energy equation, using a semi-empirical 2D MHD model of the corona to provide an empirically determined effective heat flux q_{eff} (i.e., the term effective means the possible presence of wave contributions). But, as our preliminary results indicate, in order to achieve high speed winds over the poles we also need to include the empirically determined effective pressure P_{eff} as a constraint in the momentum equation, which means that momentum addition by waves above $2 R_S$ are required to produce high speed winds. At present our calculations do not include the P_{eff} constraint. The estimates of P_{eff} and q_{eff} come from the semi-empirical 2D MHD model of the solar corona by Sittler and Guhathakurta (1999a,2002) which is based on Mk-III, Skylab and Ulysses observations. For future model development we plan to use SOHO LASCO, CDS, EIT, UVCS and Ulysses data as constraints for our model calculations. The model by Sittler and Guhathakurta (1999a, 2002) is not a self-consistent calculation. The calculations presented here is the first attempt at providing a self-consistent calculation based on empirical constraints.

INTRODUCTION

Modeling of the solar wind in coronal streamers was first attempted by Pneuman & Kopp (1971). They used an isothermal, steady state solution with an iterative technique to calculate the solar wind and the magnetic field. The iterative technique was expanded to model the steady state thermally conductive solar wind flow (e.g., Cuperman, Ofman & Dryer 1990; Stewart & Bravo, 1997). Time dependent 2D MHD models of the solar wind in a streamer have been developed by Steinolfson, Suess & Wu (1982), Wang et al. (1993), Mikic and Linker (1994), Linker and Mikic (1995), Suess et al. (1996), Wang et al. (1998). The first three-fluid 2D model was developed by Ofman (2000). Most models assume polytropic energy equations, or an ad-hoc heating function. Chen & Hu (2001) included Alfvén waves as the driving force of the wind. The stability of a 3-streamer model was investigated analytically by Wiegmann,

Schindler & Neukirch (2000). Sittler and Guhathakurta (1999b) developed a semi-empirical steady state model of the solar wind in a multipole 3-streamer structure, with the model constrained by Skylab observations. However, the semi-empirical model is not a self-consistent calculation. Here, we use the time-dependent 2D MHD equations and a multipolar initial magnetic field to obtain the self-consistent solar wind solution in a three-streamer structure. We investigate polytropic, ad hoc heating term with heat conduction and empirical heating solutions, and discuss the implications of the various approximations.

2D MHD EQUATIONS AND MODEL

The normalized resistive MHD equations are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0, \quad (1)$$

$$\rho \left[\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} \right] = -\nabla p - \frac{GM_S \rho}{r^2} + \frac{1}{c} \vec{J} \times \vec{B} + \vec{F}_v, \quad (2)$$

$$\frac{\partial \vec{B}}{\partial t} = -c \nabla \times \vec{E}, \quad (3)$$

$$\vec{E} = -\frac{1}{c} \vec{V} \times \vec{B} + \eta \vec{J}, \quad (4)$$

$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J}, \quad (5)$$

$$\frac{\partial T}{\partial t} = -(\gamma - 1) T \nabla \cdot \vec{V} - \vec{V} \cdot \nabla T + \quad (6)$$

$$(\gamma - 1)(H_c / \rho + H_i),$$

where the heat conduction by Spitzer along magnetic field lines is

$$H_c = \nabla \cdot \left(\xi T^{5/2} \frac{\nabla T \cdot \vec{B}}{B^2} \right) \cdot \vec{B}, \quad (7)$$

Here we note that Spitzer's heat conduction does not apply over the poles beyond 2 R_S . We have used uniform initial temperature. The density and the velocity were initialized by the isothermal Parker's (1963) solar wind. The boundary conditions were open at 5 R_S , and extrapolated variables with fixed $\vec{B} = \vec{B}_0$ at $r = 1R_S$. The resolution was 320 X 300, and $\eta = 10^{-4}$. We have investigated three cases for the energy equation: (A) Polytropic model with $\gamma = 1.05$, no heat conduction. (B) Heat conduction with ad hoc heat input required for the solar wind given by

$$H_i = H_0 (r - 1) e^{-r/\lambda}, \quad (8)$$

where $H_0 = 2.0$ and $\lambda = 0.7 R_S$. (C) Effective heat flux q_{eff} obtained from the semi-empirical model of Sittler and Guhathakurta (1999, 2002) on open field lines with the corresponding heat input

$$H_{i,\text{eff}} = -\vec{B} \cdot \nabla \frac{q_{\text{eff}}}{B}, \quad (9)$$

for which we note that the heat conduction is not explicitly included in case (C). Sittler and Guhathakurta (1999a) have developed a magnetic field model given by Equation (19) in that paper which uses a multipole expansion for which $\eta_M = 0.282$, $\eta_D =$

0.109, and $\eta_Q = 0.108$ are ratios of the monopole, dipole and quadropole terms relative to the octupole term, respectively. Using the radial component of the magnetic field at 1 AU they estimate the parameter $B_0 = 12$ G which is the strength of the octupole term. This magnetic field model was used in Equation (9) to calculate q_{eff} and $H_{i,\text{eff}}$ for their initial estimate for case (C). To eliminate the analytical current sheet that reaches $r = 1R_S$, and the disconnected field lines near the equator we set $\eta_M = 0$ in the magnetic field used to initialize the self-consistent MHD model. Below we present the numerical results for cases (A)-(C).

Results

For case (A) we used the following initial parameters: $\gamma = 1.05$, $B_0 = 12$ G, $T_0 = 1.6$ MK, $n_0 = 10^8 \text{ cm}^{-3}$, with the resulting normalization $V_A = 2617$ km/s, $\tau_A = 267.4$ s. For case (B) we set $\gamma = 5/3$, $B_0 = 12$ G, $T_0 = 1.4$ MK, $n_0 = 5 \times 10^8 \text{ cm}^{-3}$, with the resulting normalization $V_A = 1170.6$ km/s, $\tau_A = 598.0$. The heat conduction coefficient was $\xi = 1.1^{-3}$. Here we will not show the solutions for case (A) since they are similar to that for case (B), and rather will only show the solutions for case (B), while noting differences with case (A).

In Figure 1a we show a color display of the plasma temperature as a function of radial distance (i.e., $1 \leq r \leq 5$) and co-latitude from pole to pole for case (B). Color is used to indicate intensity. A similar format is used in Figure 1b where we show j^2/ρ . Both these figures show the presence of three helmet streamers and current sheets where in the closed field line regions the temperature can exceed 2 MK. When compared to case (A) the tops of the helmet streamers are similar and the current intensity within the current sheets is greater for case (B) when compared to case (A). These self-consistent solutions support the predictions by Sittler and Guhathakurta (1999b) that there should be three current sheets because the octupole term dominates near the Sun. Also, the wind speed at 5 R_S is 300 km/s for case (B), while for case (A) it is 250 km/s. In the model solutions by Sittler and Guhathakurta (1999a, 2002) the flow speed exceeds 400 km/s over the poles at 5 R_S . It is clear that these solutions cannot produce high speed winds over the poles as observed. Finally, in Figures 2a and 2b we show the field line topology and solar wind velocity vectors, respectively. It shows the flows are radially outward above 2-3 R_S for all latitudes. Except for the magnetic field these self-consistent solutions do not provide realistic solutions for the density, flow velocity and plasma temperature.

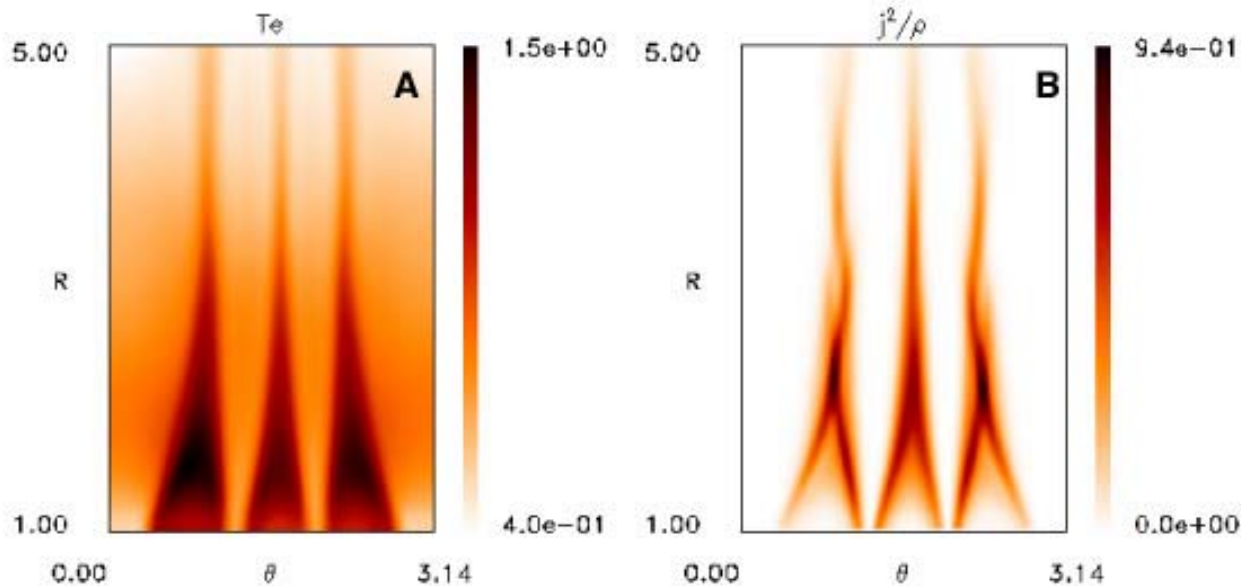


Figure 1. Color intensity maps of plasma temperature in panel A and current density squared divided by the mass density ρ in panel B. The abscissa is the co-latitude in radians and ordinate in radial distance in solar radii. The temperature is in units 1.4×10^6 °K, while j^2/ρ is in units 1.7×10^{17} statamp²/(gm-cm).

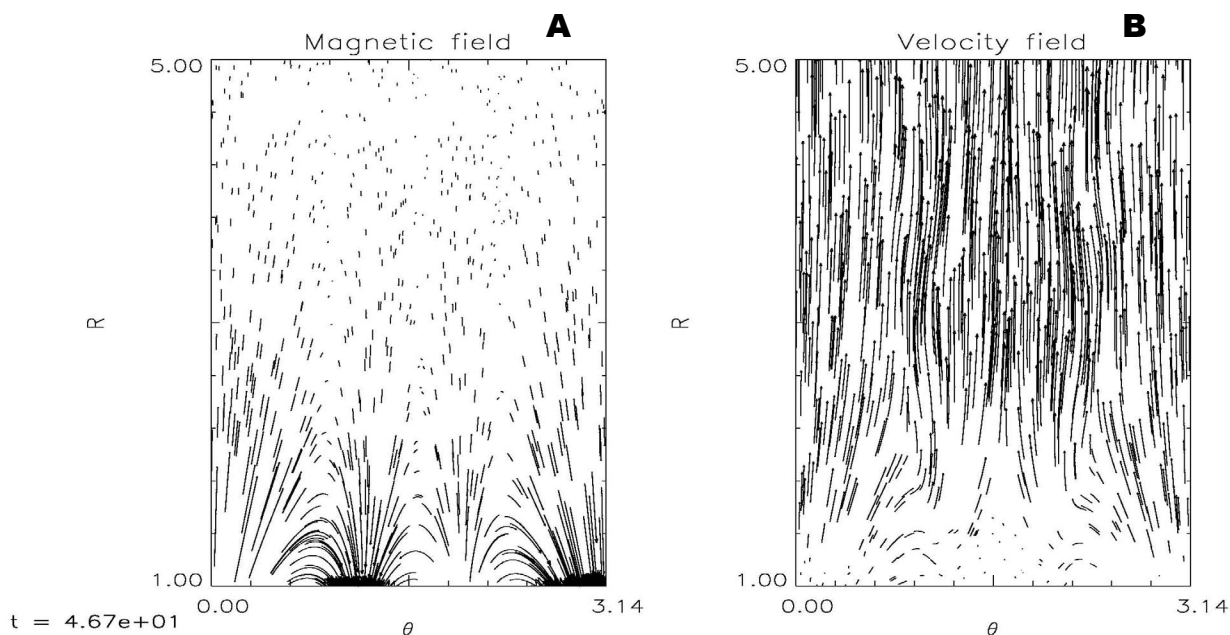


Figure 2. In panel A we show the magnetic field vectors as a function of co-latitude θ in radians and radial distance in solar radii. In panel B we show the flow velocity vectors using the same format as in panel A.

For case (C) we inserted the q_{eff} from the model by Sittler and Guhathakurta (1999a, 2002) into Eq. (9) (i.e., semi-empirical heating term) and dropped the

heat conduction term in the energy equation. For case (C) we set $\gamma = 5/3$, $B_0 = 12$ G, $T_0 = 1.4$ MK, $n_0 = 5 \times 10^8$ cm⁻³, with the resulting normalization $V_A = 1170.6$

km/s, $\tau_A=598.0$. The normalization for q_{eff} given in $\text{erg/cm}^2/\text{s}$ was 1.13×10^7 . In Figure 3 we show the flow speed over the poles and compare it to a Parker solution. Inside of $2 R_S$ the flow speed exceeds the Parker solution and then drops below the Parker solution at greater heights. We believe this result is caused by not including momentum addition terms due to waves. We feel this will be corrected once P_{eff} is

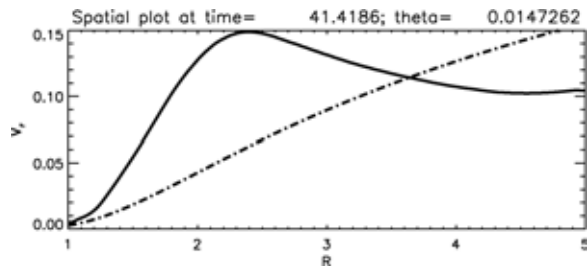


Figure 3. The flow speed over the poles for case (C), solid line, in units of $V_A=1171$ km/s. For reference we show Parker solution as dash-dot line.

used as a constraint in the solutions and will get high speed flows over the poles. This should also allow us to acquire a semi-empirical estimate of the momentum addition due to waves. Finally, the use of q_{eff} in the energy equation will tend to over-estimate the gas temperature since we are not able to separate wave contributions at this time.

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