Revised method for calculating cloud densities in equilibrium models

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Overview

We discuss a correction to the calculation of cloud density in equilibrium cloud condensation models (ECCMs) based on Weidenschilling and Lewis (1973). All other quantities, such as lapse rate and gas mixing ratios, are not affected by this correction.

Although more complex 2D and 3D simulations are preferred to model planetary and exoplanetary atmospheres, ECCMs are still used by many researchers to quickly estimate wet adiabatic atmospheric structures, saturated vapor profiles, and cloud base levels. With the conceptual corrections presented here, cloud densities can also be estimated.

Numerical revisions

Weidenschilling and Lewis (1973; hereafter WL73) give a relation for calculating cloud density, based on differencing the condensate column density between adjacent model layers:

$$D = \rho H \sum_{\kappa} \frac{\mu_\kappa}{\bar{\mu}} \left(\frac{X_{\kappa I} - X_{\kappa J}}{\Delta} \right) \quad [1]$$

This equation gives a result in units of density, which is mass/volume. However, the volume for the calculation has unit area and a height of *H*. We derive a similar relation, which differs from the WL73 formula by a factor of *L*/*H*, with *L* = unit length, which gives a standard density based on the mass per unit volume (see Fig. 1):

$$D = \rho L \sum_{\kappa} \frac{\mu_\kappa}{\bar{\mu}} \left(\frac{X_{\kappa I} - X_{\kappa J}}{\Delta} \right) \quad [2]$$

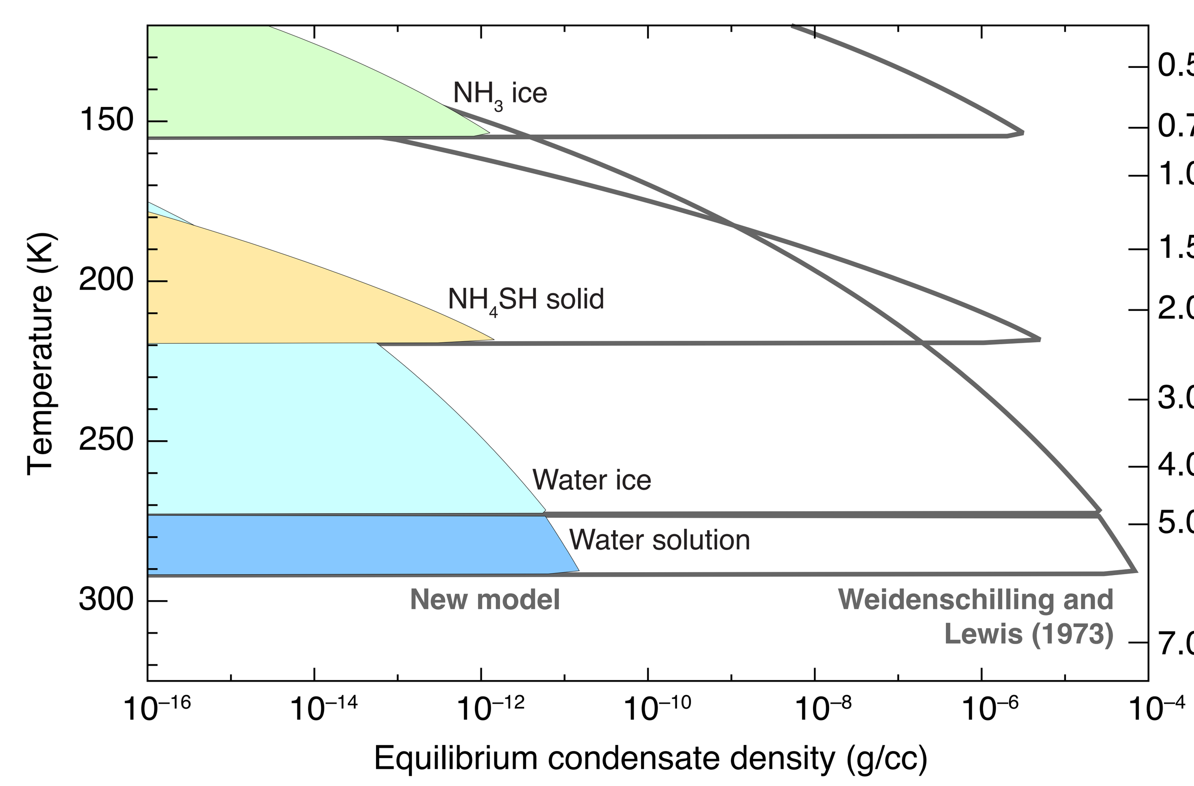

Fig. 1: Numerical revisions to the equilibrium cloud model. Comparison of equilibrium condensate densities calculated with Eqn. 1, from WL73 (grey outlines), and with Eqn. 2 (solid fills). Densities with the new equation are lower by a factor of *L*/*H*.

Conceptual revisions

We call densities calculated by ECCMs "equilibrium condensate densities," to distinguish from actual cloud densities. Conceptually, the equilibrium condensate density $\rho_{equilib.}$ describes the amount of condensate produced when a unit volume of saturated air rises a unit distance upward, wet adiabatically (Fig. 2). But real clouds do not form in a perturbed closed system; the actual displaced volume depends on the duration and velocity of the updraft, and is modified by microphysical processes such as precipitation and evaporation.

Maintaining the concept from WL73, that condensed material remains at the altitude where it condenses out, we can calculate first order cloud densities by multiplying $\rho_{equilib.}$ by the updraft speed and duration, and dividing by the unit length to convert time-integrated mass flux to mass density:

$$\rho_{cloud} = \frac{\rho_{equilibrium} \; w \; t_{draft}}{L} \quad [3]$$

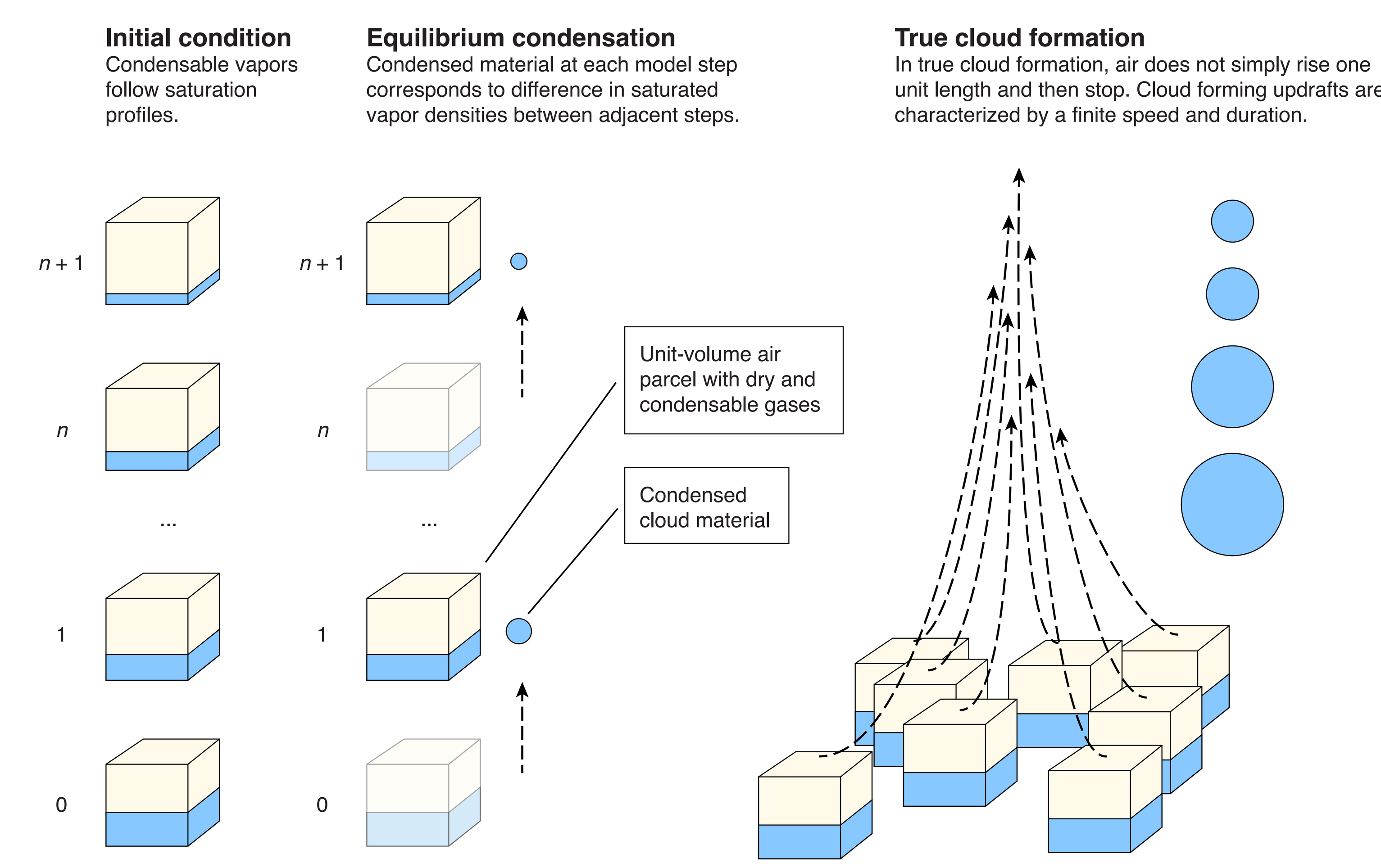

Fig. 2: Conceptual revisions to the equilibrium cloud model. **Left column:** Initial condition is a saturated vapor mixing ratio profile and wet adiabatic temperature gradient. **Center column:** In the ECCM, a unit volume of moist air rises a unit distance, producing the equilibrium condensate density $\rho_{equilib.}$. **Right column:** To estimate cloud density, $\rho_{equilib.}$ must be multiplied but the updraft speed and duration to give an integrated flux (Eqn. 3).

Validation

We attempt to validate the cloud density algorithm by comparing model data with cloud observations on Jupiter and on Earth.

Jupiter: In Atreya et al. (1999), we compared equilibrium cloud densities to the Galileo Probe Nephelometer measurements of cloud mass loading (Ragent et al. 1998) in the probe entry site (PES) on Jupiter. Table 1 compares the new and old model cloud mass loading with the PES measurements. Old model mass loadings (calculated with Eqn. 1) are much greater than observations, so precipitation or other processes would need to be invoked, which may not be reasonable considering the thin clouds thought to be present in 5-μm hot spots. But the new algorithm gives much smaller densities, that could be could be consistent with observations depending on updraft characteristics. Simulations of Jupiter-like atmospheres have vertical velocities in the range of 10s of m s^{-1} for convective plumes (Nakajima et al. 2000, Hueso and Sánchez-Lavega 2001), cm s^{-1} for parameterized convection (Del Genio and McGrattan 1990), or mm s^{-1} for spatially averaged global circulation (Zuchowski et al. 2009a,b). An updraft of 1 cm s^{-1}, in the middle of the range of simulated vertical velocities, could match some PES cloud column densities for very reasonable updraft durations of 100–1000 s.

	Galileo Probe Nephelometer (PES)	Old model (PES height)	New model (PES height)	Old model (full height)	New model (full height)
Water cloud properties					
P (bar)	2.45–3.58	2.45–3.58	2.45–3.58	5.96	5.96
T_{base} (K)	247	250	250	292	292
ρ_c (g cm^{-2})	1.1–3.2×10^{-5}	2.4	6.1×10^{-7}	61.8	1.4×10^{-5}
NH_4SH cloud properties					
P (bar)	1.0–1.34	1.0–1.34	1.0–1.34	2.32	2.32
T_{base} (K)	176	185	185	219	219
ρ_c (g cm^{-2})	7.6–130×10^{-5}	3.2×10^{-4}	1.1×10^{-10}	1.56	4.5×10^{-7}
NH_3 cloud properties					
P (bar)	0.46–0.53	0.46–0.53	0.46–0.53	0.78	0.78
T_{base} (K)	138	136	136	155	155
ρ_c (g cm^{-2})	0.3–5.7×10^{-5}	2.6×10^{-2}	1.2×10^{-8}	1.13	4.7×10^{-7}

Table 1: Observed and modeled Jupiter cloud properties. Observed cloud properties are from Ragent et al. (1998) for the Galileo Probe Entry Site (PES). Model output columns are integrated either over the same pressure ranges as the observed clouds (where labeled "PES height") or over the full model height. Values labeled "old model" are for $\rho_{equilib.}$ calculated with Eqn. 1; "new model" is $\rho_{equilib.}$ calculated with Eqn. 2.

We originally investigated corrections to the WL73 cloud density formula based on comments by Andy Ingersoll that the densities violated conservation of mass. To test this, Fig. 3 compares finite column densities integrated upwards starting from the modeled water cloud base, to the total precipitable water in a unit volume at the cloud base (horizontal dashed line). According to the formalism of the ECCM, the total mass loading should not exceed the dashed line, because all the condensed vapor should have come from the uplifted parcel at cloud base. The corrected model asymptotically approaches the dashed line, indicating that mass is conserved. The WL73 formula violates conservation of mass by a factor of *H*/*L*.

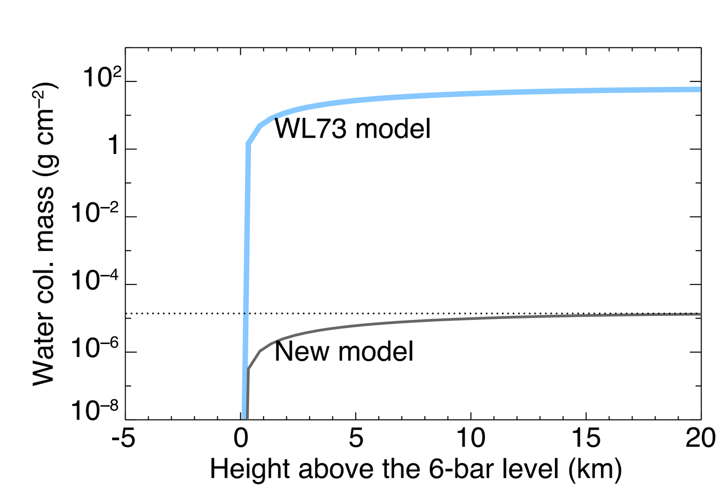

Fig. 3: Cumulative water cloud column mass, integrated upward from 6 bar. Dotted line is the total precipitable water in a unit volume at cloud base. The old condensate density formula (Eqn. 1) exceeds the dotted line and thus violates conservation of mass.

Earth: Figure 4 shows $\rho_{equilib.}$ for an Earth atmosphere case. We can use Eqn. 3 to convert the cloud base $\rho_{equilib.}$ of ~10^{-11} g cm^{-3} to cloud density, as a function of updraft speed and duration. Figure 5 shows typical updraft characteristics. Gold shading shows estimated cirrus updraft velocities range from 10–20 cm s^{-1} (Houze 1993) to 2 cm s^{-1}, and persist for around 10–15 min (Starr and Cox 1985). Blue shading, for non-precipitating cumulus, covers the range of 1–5 m s^{-1} and durations of 15–25 min (Houze 1993, Mansell et al. 2005).

The corresponding cloud densities (see caption and *z*-axis shading) compare very well with observed cloud densities and column densities: cirrus densities range from 3x10^{-9} to 4x10^{-7} g cm^{-3} but cluster around 10^{-8} g cm^{-3} based on radar and aircraft measurements (Deng and Mace 2006, Shupe et al. 2002, Lawson et al. 2002). Cumulus cloud water content is 4x10^{-6} g cm^{-3} (Mansell et al. 2005).

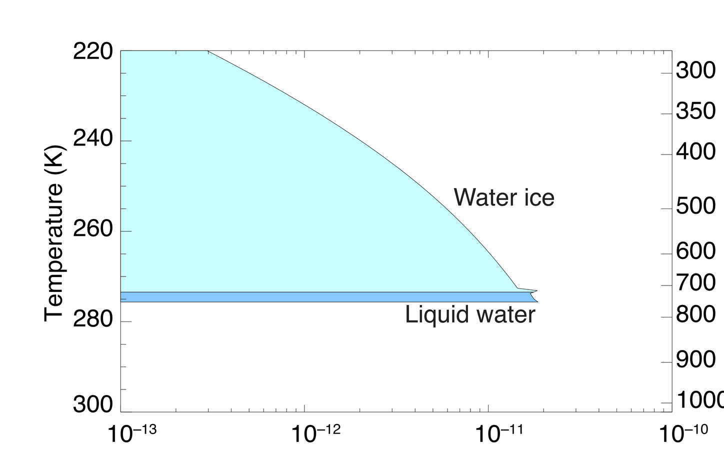

Fig. 4: New model output densities for a terrestrial case, with an initial condition of 1% water volume mixing ratio at 100 mbar. This gives the equilibrium condensate density at each level, equivalent to material condensed in a unit velocity updraft (1 cm s^{-1}) persisting for 1 time unit (1 sec). To convert to an actual cloud density, use Fig. 5 and Eqn. 3.

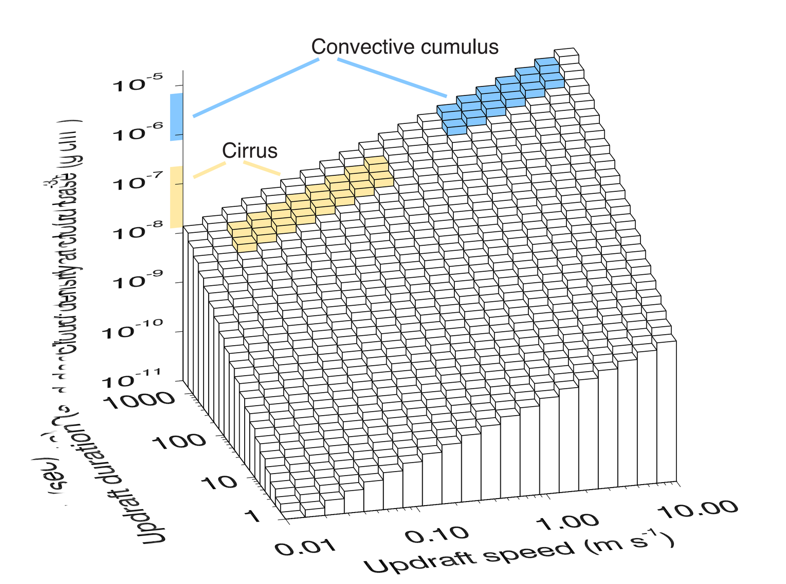

Fig. 5: Estimated cloud density (*z*-axis) as a function of updraft speed and duration (*x*- and *y*-axes), for the terrestrial cloud case. The lower left corner—at unit updraft velocity and duration—is the value of the equilibrium condensate density plotted in Fig. 4. For reasonable updraft characteristics (shaded surfaces; see text), modeled cloud densities for cirrus are 1.3–20 ×10^{-8} g cm^{-3} (or 0.013–0.2 g m^{-3}), and for cumulus are 8–70 ×10^{-6} g cm^{-3} (or 0.8–7 g m^{-3}).

References/symbols

Atreya, S.K. et al. (1999) *PSS* 47, 1243–1262.
Deng, M.and G.G. Mace (2006) *J. Appl. Met. Climatology* 45, 1690–1709.
Lawson, P. (2002) CRYSTAL-FACE team meeting, Feb. 24-28, Salt Lake City, UT.
Shupe, M. et al. (2002) CRYSTAL-FACE team meeting, Feb. 24-28, Salt Lake City, UT.
del Genio, A.D. and K.B. McGrattan (1990) *Icarus* 84, 29–53.
Mansell, E.R. et al. (2005) *JGR* 110, D12101 (24 pp.).
Nakajima, K. et al. (2000) *GRL* 27, 3129–3132.
Houze, R.A. (1993) *Cloud dynamics*. Academic Press, Boston MA.
Hueso, R. and A. Sánchez-Lavega (2001) *Icarus* 151, 257–274.
Ragent, B. et al. (1998) *JGR* 103, 22891–22910.
Starr, D.O. and S.K. Cox (1985) *JAS* 42, 2663–2681.
Weidenschilling, S.J. and J.S. Lewis (1973) *Icarus* 20, 465–476.
Zuchowski, L.C. et al. (2009a,b) *Icarus* 200, 548–573.

ρ = atmospheric density (g cm^{-3})
$\rho_{equilib.}$ = cloud model output; equilibrium condensate density (g cm−3)
ρ_{cloud} = cloud density (g cm^{-3})
ρ_c = column density (g cm^{-2})
μ = molecular weight (g mol^{-1})
Δ = model step size (cm)
D = WL73 notation for $\rho_{equilib.}$
H = atmospheric scale height (cm)
X = condensate gas volume mixing ratio
L = unit length (cm)
w = updraft velocity (cm s^{-1})
t_{draft} = updraft duration (s)