PREDICTIONS OF THE FEATURES FOR SUNSPOT CYCLE 23

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Abstract. Smoothed monthly mean Ap indices are decomposed into two components $(Ap)_c$ and $(Ap)_n$. The former is directly correlated with the current sunspot numbers, while the latter is shown to achieve its maximum correlation with the sunspot numbers after some time lag. This latter property is used to develop a method for predicting the sunspot maximum based on the observed value of $(Ap)_n$ maximum which occurs during the preceding cycle. The value of R_M for cycle 23 predicted by this method is 149.3 ± 19.9. A method to estimate the rise time (from solar minimum to maximum) has been developed (based on analyses of Hathaway, Wilson, and Reichmann, 1994) and yields a value of 4.2 years. Using an estimate that the minimum between cycles 22 and 23 occurred in May 1996, it is predicted that the sunspot maximum for cycle 23 will occur in July 2000.

1. Introduction

The cyclic behaviour of solar activity, in particular the 11-year sunspot cycle, is now a well-known, if imperfectly understood, property of the Sun. Apart from providing information of the physical process inside the Sun, solar activity gives rise to the variations of the solar-terrestrial environment, which may affect radio communication, the orbital lifetime of low Earth-orbit satellites, and weather patterns. Thus much effort has been devoted to forecasting the properties of future cycles, in particular, the amplitude and phase of sunspot maximum. Prediction methods are based either on historical sunspot numbers or on an assumed understanding of the physical processes involved.

Periodicities or regularities present in historical sunspot numbers have been used for predictions (Hathaway, Wilson, and Reichmann, 1994; Wilson, 1992). Recently many authors studied the chaotic nature of the solar activity cycle (Mundt, Maguire, and Chase, 1991; Wilson, 1994; Feynman and Gabriel, 1990), and predictions were also attempted assuming such chaotic behaviour (Kurths and Ruzmaikin, 1990).

Predictions based on physical processes generally invoke the Babcock model of the cycle. Although it is no longer widely accepted, the model suggests that the poloidal magnetic field at sunspot minimum is found by differential rotation into a subsurface toroidal field, loops of which emerge as the next cycle's active regions. Therefore some predictions based on physical modelling assume that the strength of the polar magnetic field at sunspot minimum provides a prediction for the amplitude of the next sunspot cycle (Schatten *et al.*, 1978; Schatten and Sofia, 1987). Unfortunately the polar magnetic field is difficult to measure accurately, and other phenomena such as the number of polar faculae or the shape of the corona at eclipses have been proposed as proxies for the polar fields.

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Whether or not these phenomena are genuine proxies for the polar fields, they and some other physical quantities, such as high latitude ephemeral active regions, minimum sunspot number and geomagnetic indices, have been regarded as precursors of the next sunspot cycle. The use of precursors as predictors of the properties of the cycle implies that the (unknown) physical processes responsible for the cycle also give rise to related activity phenomena prior to the emergence of the first sunspots of a new cycle. For example, it has been proposed that the bipolar ephemeral regions, which emerge at high-latitudes during the declining phase of the old cycle form part of an extended cycle (Wilson *et al.*, 1988) which begins emerge about or before the maximum of the old cycle.

However, without a clear understanding of the physical processes involved, the use of precursors must depend on their success in predicting the properties of previous cycles. A particular benefit of precursor predictions is that if a particular precursor leads to consistently successful predictions, important physical connections may be indicated.

While the validity of precursors as predictors may be tested against data from past cycles, the only significant results come from the success (or otherwise) of predictions made before the beginning of a new cycle. In cycle 22, predictions based on the geomagnetic indices, *aa* and Ap, proved to be the most successful (Thomson, 1988, 1993; Kane, 1989; Gonzalez and Schatten, 1987; Layden *et al.*, 1991) and, since the first sunspots of the new cycle (cycle 23) have just been reported (May 1996, McIntosh, private communication), the time would appear to be appropriate to attempt predictions regarding the properties of cycle 23.

In this paper, we propose an approach using a geomagnetic precursor method, based on the geomagnetic Ap indices, to predict the maximum sunspot number, R_m, of the coming cycle 23. The predicted value of R_m is then used to determine the parameters of a function that attempts to fit the time series of the sunspot numbers (Hathaway, Wilson, and Reichmann, 1994). This leads to a prediction of the rising time from sunspot minimum to maximum.

2. Feynman's Approach

The relation between geomagnetic activity and solar activity has been studied by Feynman (1982) and many other authors. A natural picture of the relation is that the geomagnetic activities are affected and controlled by the interplanetary magnetic field (IMF) and the solar wind. The daily aa and Ap geomagnetic indices are measures of the responses of geomagnetic field to variations in the IMF and the solar wind. The aa-index is the average of the measurements at two antipodal stations (Melbourne, Australia, and Greenwich, UK). The Ap-index is the average of the measurements at 13 observatories distributed around the globe (National Geophysical Data Centre ftp site). When studying the geomagnetic and solar wind cycle, Feynman (1982) defined the lower envelope of the data points in the scatter

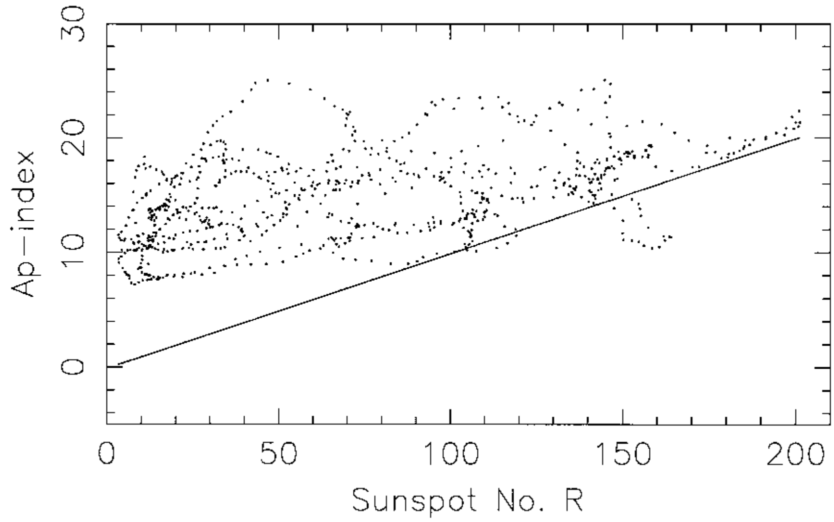

Figure 1. Smoothed monthly mean Ap indices from 1932 to 1994 against smoothed monthly mean sunspot numbers. The solid line correspnds to Equation (3).

plot of the annual mean geomagnetic Aa indices against the sunspot numbers from the years 1869 to 1975 to be a straight line, which was later called the *extreme locus* (Thompson, 1985). Annual mean geomagnetic indices Aa were thus decomposed to two components, one of which was strictly proportional to the current sunspot cycle, with the coefficient being the gradient of the manually defined extreme locus; the other was the remaining values of the indices after arithmetically substracting the values of the extreme locus. Feynman claimed that the first component is related to the short-lived solar activity and the second to long-lived features such as coronal holes. The first component has, by definition, a correlation of 1 and is in phase with the sunspot numbers. We found a high correlation between the second component and the sunspot numbers, although they are out of phase by $\sim 180°$.

In next section, following Feynman's idea, but applying a new method on a data set extending up to the present, we decompose smoothed monthly mean geomagnetic Ap indices into two components, and explore the relation between one of these and sunspot numbers. We then use this component of Ap to predict the next sunspot cycle maximum.

3. The Prediction Method

Daily sunspot numbers and Ap indices from January 1932 to April 1996 were obtained from the National Geophysical Data Center (NGDC), NOAA ftp site. Smoothed monthly mean sunspot numbers provided by NGDC were obtained as the arithmetic average of two sequential 12-month running mean of monthly mean

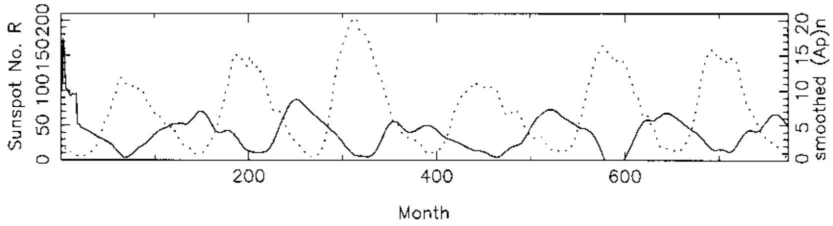

Figure 2. Smoothed monthly mean sunspot numbers (dotted line) and smoothed $(Ap)_n$ (solid line).

numbers and the same procedure is applied to the Ap indices to get the corresponding smoothed monthly means. Figure 1 shows the scatter plot between the smoothed monthly mean Ap indices and the sunspot numbers. We now decompose Ap indices into two components. One of the components $(Ap)_c$ is defined to be a linear function of the sunspot number R, i.e.,

$$(Ap)_c = a + bR \,, \tag{1}$$

where R is the smoothed monthly mean sunspot number, and a and b are unknown constants. The other component is then

$$(Ap)_n = Ap - (Ap)_c \,. \tag{2}$$

To determine a and b, we first set $a = 0$ and vary b by increments of 0.01 to get the maximum cross-correlation between R and $(Ap)_n$, allowing for a variable phase difference. Then we vary a again by increments of 0.01 to get the maximum correlation using this value of b. The iterative process is repeated until the optimal values of a and b are unchanged. This procedure yields $a = -0.11$ and $b = 0.10$, and Equation (1) becomes

$$(Ap)_c = -0.11 + 0.10R \,. \tag{3}$$

This relation is represented by the solid line in Figure 1, while the dots represent smoothed monthly mean Ap indices.

Since the values of $(Ap)_n$ obtained directly from Equation (2) are rather noisy, we applied a 35-month running average before calculating the cross correlation to determine a and b. The maximum correlation between R_m and $(Ap)_n$ achieved was 0.90 with a time lag for R_m of 60 months or 5 years (implying that $(Ap)_n$ is the precursor). The smoothed $(Ap)_n$ and the smoothed monthly mean sunspot number as functions of time are shown in Figure 2 as a solid line and a dotted line, respectively.

The maximum amplitude of the smoothed $(Ap)_n$ occurs during the declining phase of each sunspot cycle, or at sunspot minimum. The maxima of $(Ap)_n$ and the years of the maxima are shown in Table I, as well as the years of the nearest sunspot

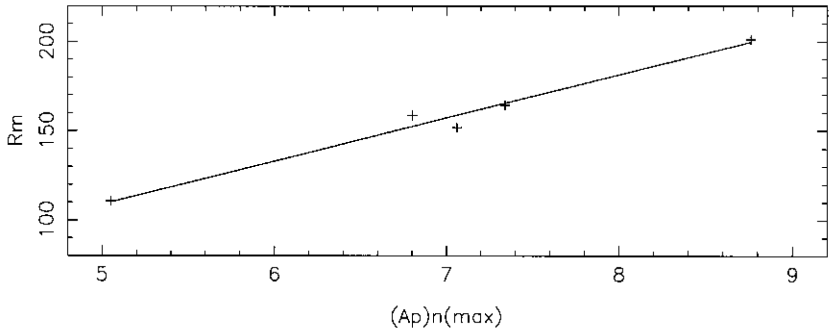

Figure 3. $(Ap)_n$ maxima against next cycle sunspot maxima, and the best fit by linear regression.

Table I
Observed and predicted maximum sunspot numbers and related information

Cycle No.	Year of sunspot max.	$R_M(O)$	$R_M(P)$	ΔR	$(Ap)_{n(\max)}$	Year of $(Ap)_{n(\max)}$	Year of sunspot min.
18	1947.5	151.8	158.7 ± 20.5	6.9	7.1	1944.4	1944.2
19	1957.9	201.3	199.6 ± 23.1	-1.7	8.8	1952.9	1954.3
20	1968.9	110.3	110.2 ± 15.2	-0.1	5.0	1964.5	1964.9
21	1979.9	164.5	165.4 ± 20.8	0.9	7.3	1975.4	1976.5
22	1989.6	158.5	152.4 ± 20.1	-6.1	6.8	1984.7	1986.8
23	?	?	149.3 ± 19.9	?	6.7	1995.2	?

minima. There are 2 maxima in $(Ap)_n$ before the sunspot minimum between cycles 19 and 20. Similar problems were encountered for different cycles in Kane (1989, 1992) and Layden *et al.* (1991) when relating aa and Ap minima to the next cycle sunspot maxima. Here we choose the second peak, which gives a slightly better fit than the first peak value to the linear regression described below.

A linear regression analysis between the $(Ap)_n$ maxima and the next sunspot maxima R_M was carried out, yielding Equation (4) with the number of points $n = 5$ and correlation coefficient $r = 0.99$:

$$R_M = (-11.53 + 14.52) + (24.11 \pm 2.04)(Ap)_{n(\max)} . \tag{4}$$

The 5 data points and the fitted straight line (Equation (4)) are shown in Figure 3. The 'predicted' maximum sunspot numbers $R_M(P)$ as derived from (4) using the values for $(Ap)_{n(\max}$, the observed maximum sunspot number $R_M(O)$, and the difference between predictions and observations ΔR for cycles 18 onwards are given in Table I.

As can be seen from the correlation coefficient of the regression and Table I, this is a good fit and, for last 5 cycles, the 'predicted' values shown in column 4 are very close to observed values in column 3, the largest difference being 6.9 for cycle 18.

It may be noted that the ‘predicted’ values of R_M for previous cycles in Table I have been based on data extending to April, 1996, i.e., on data from the later cycles for which the predictions were made. This differs from the real sense of predictions. For this reason and to further test the method, we exclude the data from September 1986, just before the last solar minimum, and repeat the entire procedure to predict the maximum sunspot number for cycle 22, and compare with the observed value. For this data set, the correlation obtained between $(Ap)_n$ and the sunspot number R is 0.89 for $a = -3.55$, $b = 0.12$ (in Equation (1)), with the time lag of R being 59 months. The predicted maximum sunspot number for cycle 22 is 150.6 ± 35.8 which is less by 7.9 than the observed value, but very close to 152.4 ± 20.1 the ‘predicted’ value for cycle 22 using the larger data set. Therefore this method is successful for predicting the maximum sunspot number of cycle 22, and the calculations are not very sensitive for different data set.

On the basis of this method, our prediction of R_M for cycle 23 is 149.3 ± 19.9.

4. Other Features of Cycle 23

We now attempt to predict other features of the cycle such as the shape of the distribution and the rise time from the minimum to the maximum. Hathaway, Wilson, and Reichmann (1994) proposed a function to fit each cycle of the sunspot number time series of the form

$$f(t) = a(t - t_0)^3/\{\exp[(t - t_0)^2/b^2] - c\} \ , \tag{5}$$

where t_0 is the time of the commencement of each cycle, and a, b, and c are parameters, which were determined for each cycle by a nonlinear least-square method (Hathaway, Wilson, and Reichmann, 1994). They found that a single value $c = 0.71$ is adequate for all 21 cycles. The time of the sunspot maximum was given by the value of $t = t_m$ for which $\mathrm{d}f/\mathrm{d}t = 0$, i.e.,

$$t_m = t_0 + 1.081b \ , \tag{6}$$

where t_m is the time of sunspot maximum. The maximum sunspot number (being $f(t_m)$) is given by

$$R_M = 0.504ab^3 \ . \tag{7}$$

Hathaway, Wilson, and Reichmann also found a statistical relation between a and b,

$$b = 27.12 + 25.15/(a \times 10^3)^{(1/4)} \ . \tag{8}$$

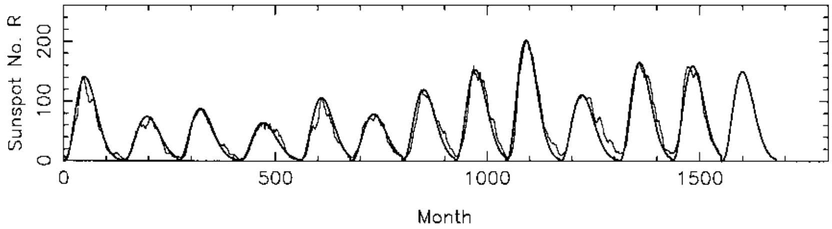

Figure 4. Monthly mean sunspot numbers (thin line) and fitted curve by function (6) with the predicted cycle 23 (thick line).

In the analysis given by Hathaway, Wilson, and Reichmann (1994), parameters a and b for a new cycle were first determined at some time after the beginning of a sunspot cycle, and R_m was then given by (8). Here, we make use of the predicted R_m in Section 3, and solve (8) and (9) to determine a and b, and therefore obtain $\delta t = t_m - t_0$, the time (measured in month) from cycle minimum to maximum.

Using our predicted R_M, i.e., 149.3 $\pm$ 19.9, we obtain $a = (3.0 \pm 0.8) \times 10^{-3}$ and $b = 46.2 \pm 1.6$, and $\delta t = 50.0 \pm 1.8$ month. The rise time of cycle 23 thus predicted is 4.2 $\pm$ 0.2 years.

We also used the observed R_M values for the previous cycles to obtain the corresponding values of a and b in Equation (6) for each cycle. Then the corresponding function $f(t)$ for each cycle from cycle 11 is plotted together with smoothed monthly mean observed sunspot numbers in Figure 4. For most of these cycles, it is seen that although the only information supplied is the maximum sunspot number for each cycle, the functions follow the observations fairly closely, in particular the times of the maxima agree well. We thus have some confidence in the predicted value of the rise time for cycle 23 given by this method. The preliminary estimate of the sunspot minimum, as mentioned in the Introduction, is May 1996 (McIntosh, private communication) by examining the appearance of the old and new cycle bipole regions. If May 1996 will be proved correct, the next sunspot maximum should occur in July 2000 with $\pm$2 months uncertainty.

5. Discussions and Conclusions

A precursor method based on the smoothed monthly mean geomagnetic Ap indices has been proposed and applied to predict the maximum sunspot number R_M for cycle 23. A linear relation with the correlation as high as $r = 0.99$ for 5 data points was achieved between the maxima of $(Ap)_n$ and the maxima of next sunspot cycles. Predictions can be made by this method at or slightly before the sunspot minimum (see column 7 in Table I), while most of the other methods using aa or Ap indices give predictions either at the minimum or some years after.

A few predictions for cycle 23 have already been published, and these are summarized in Table II, together with the results obtained from the method described

Table II
Published predictions for cycle 23

Author	Method	Predicted R_m
Schatten and Pesnell (1993)	solar dynamo method	170 ± 25, at 1999.7 ± 1 year
Wilson (1992)	even-odd pair	198.8 ± 36.5 or 213.9 ± 37.5
Letfus (1994)	a bimodal behaviour of	195.1 ± 17.1, rise time
		3.7 ± 0.5 year
	even-odd sunspot cycle pairs	or 181.3 ± 44.3
Obridko (1994)	even-odd pair rule	203.2 ± 10.7
	an index $p = \partial Aa/\partial R$ during	
	sunspot declining phase	74.7 ± 6.9
Thompson method (private	number of geomagnetic disturbed	
communications, using data	days measured by Ap indices	≥ 164.9
till April 1996)		
This paper	one component of	
	monthly mean Ap indices	149.3 ± 19.9, rise time
		4.2 ± 0.2 year

in Sections 3 and 4. Most of these predicted values are greater than that obtained by this method. Amongst the methods, those based on even-odd pair rule naturally predict that cycle 23 will be of greater size than cycle 22. Our prediction of R_M for cycle 23 is slightly less than the sunspot maximum number of cycle 22, although the difference is not significant within our stated errors.

By this method the R_m for cycle 23 is predicted to be 149.3 ± 19.9, and should occur 4 years and 2 months after the minimum between cycles 22 and 23.

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